An Inventive Method For Solving Fully Interval Transportation Problem

G. Padma karthiyayini[1], Dr.S. Ananthalakshmi[2], Dr. R. Usha Parameswari[3]

[1]Research Scholar (Reg. No:18222072092001), [2] Assistant Professor, [3]Assistant Professor
Department of Mathematics,
[1]Govindammal Aditanar College for women,Tiruchendur,
Affiliated to manonmaniamSundaranar University, Tirunelveli-627 012,India.
[2]AnnaiHajira Women's College, Melapalayam,Tirunelveli-627 012,India.
[3]Govindammal Aditanar College for Women,Tiruchendur.

Abstract

This paper discusses, the transportation problem (TP) under uncertainty, particularly when parameters are given in interval forms, is formulated. That is the shipping cost, supply and demand parameters are all intervals. And also presents the interval parameters would seem to monitor the capability of fixed charge transportation problem. Furthermore, the solution of the interval transportation problem (ITP) is analyzed.

Keywords: *Transportation problem (TP), fixed charge transportation problem, Interval number, interval transportation problem (ITP)*

Introduction

The transportation problem (TP) is one of the optimization problems in which objective is to transport at the optimal distribution of the various quantities from several sources to different destinations in such a way that the total transportation cost is minimum. In general, a traditional transportation model consists of an objective function and two kinds of constraints, namely source constraint and destination constraint. Itwasoriginated by Hitchock[1] in 1941, concerning its special structure, for finding optimal solutions to TP different methods are discussed in many papers[2,3] and so far. Chanas et.al [4] discussed possible cases of TP with interval parameter and fuzzy parameters. The fixed charge problem was founded by Hirsch and Dantzing [5] in 1954. Solving the interval transportation problem, researchers have divided the problem into two sub-problems namely, upper and lower level. Firstly, the upper level problem is solved and after that, the lower level problem with upper bound constraints on the decision variables is solved.Sengupta and pal [6] presented a new fuzzy orientation method for solving interval TPs by considering the midpoint and width of the interval in the objective function.A.Akilbashaet.al [7] discussed the usage of mid-width method for independent ITP. M.R.Safi, A.Razmjoo[8]developed two different order relations for interval numbers, two solution procedures. S.M. AbulKalamAzad[9] developed algorithm for the average of total opportunity costs of cells along each row identified as Row Average Total Opportunity Cost (RATOC) and the average of total opportunity costs of cells along each column identified as Column Average Total Opportunity Cost (CATOC).

This paper is structured as follows: In section 2, some basic definition and results were related to real intervals are presented. The next section is discussedan interval TP. In addition, appropriate procedure for fixed cost TP is discussed. In section 4, average total opportunity cost methodis used. Succeeding section a numerical example is given for understanding the solution procedure of the proposed method and finally, the conclusion is given in section 5.

2. Preliminaries

The aim of this section is to present some notations, notions and results which are of useful in our further consideration.

Let us denote by I the class of all closed and bounded intervals in *R*. If [*a*], [b] are closed and bounded intervals, then the notation [*a*] = [$\underline{a}, \overline{a}$] and [b] = $[\underline{b}, \overline{b}]$, where $\underline{a}, \underline{b}$ and $\overline{a}, \overline{b}$ mean the lower and upper bounds of [*a*], [b]. Let [*a*] = [$\underline{a}, \overline{a}$] and [b] = $[\underline{b}, \overline{b}]$ be in I. Then by definition,

(i) [*a*] + [b] = [$\underline{a}+\underline{b}, \ \overline{a}+\overline{b}$] ∈ I

(ii) [*a*] - [b] = [$\underline{a}-\overline{b}, \ \overline{a}-\underline{b}$] ∈ I

(iii) - [a] = [$-\overline{a}, \ -\underline{a}$] ∈ I

(iv) $x\,[\underline{a}, \overline{a}\,] = [x\underline{a}, x\overline{a}\,]$, if $x \geq 0$

$[x\overline{a}, x\underline{a}\,]$, if $x \leq 0$

Where *x* is a real number.

(v) An interval [*a*] is said to be positive, if $\underline{a} > 0$ and negative, if $\overline{a} < 0$.

(vi) If [*a*] = [$\underline{a}, \overline{a}$] and also [b] = $[\underline{b}, \overline{b}]$ are bounded and real intervals, consider the multiplication of two intervals as follows:

[*a*][b] = [min { $\underline{ab}, \ \underline{a}\overline{b}, \ \overline{a}\underline{b}, \ \overline{ab}$ }, max { $\underline{ab}, \ \underline{a}\overline{b}, \ \overline{a}\underline{b}, \ \overline{ab}$ }]

1) If $0 \leq \underline{a} \leq \overline{a}$ *and* $0 \leq \underline{b} \leq \overline{b}$ then write

$$[a][b] = \left[\underline{ab}, \overline{ab}\right]$$

2) If $0 \leq \underline{a} \leq \overline{a}$ *and* $\underline{b} < 0 < \overline{b}$ then write

$$[a][b] = \left[\underline{a}\overline{b}, \ \overline{ab}\right]$$

3) If $\underline{a} \leq \overline{a} \leq 0$ *and* $0 \leq \underline{b} \leq \overline{b}$ then write

$$[a][b] = \left[\overline{a}\underline{b}, \ \underline{a}\overline{b}\right]$$

4) If $\underline{a} \leq \overline{a} \leq 0$ *and* $\underline{b} < 0 < \overline{b}$ then write

$$[a][b] = \left[\overline{a}\underline{b}, \ \underline{ab}\right]$$

(vii) There are several approaches to define interval division. Consider the quotient of two intervals as follows: Let [*a*] = [$\underline{a}, \overline{a}$] and also [b] = $[\underline{b}, \overline{b}]$ be two nonempty bounded real intervals. Then if $0 \notin [\underline{b}, \overline{b}]$ write

$$[a]/[b] = [\underline{a}, \ \overline{a}]\left[\frac{1}{\overline{b}}, \frac{1}{\underline{b}}\right]$$

(viii) For an interval [a] such that $\underline{a} \geq 0$, define the square root of [*a*] denoted by $\sqrt{[a]}$ as: $\sqrt{[a]}$ = { $\sqrt{b} : \underline{a} \leq b \leq \overline{a}$ }.

(ix) Mid-point of an interval [a] is defined as $m([a]) = \frac{1}{2}\left(\underline{a} + \overline{a}\right)$.

(x) Width of an interval [a] is defined as $w([a]) = \overline{a} - \underline{a}$.

(xi) Half-width of an interval [a] is defined as $w([a]) = \frac{1}{2}(\overline{a} - \underline{a})$.

Remark 2.1: Every real number $a \in R$ can be considered as an interval $[a, a] \in$ I.

Definition 2.1: The function F: $R^n \to$ I defined on the Euclidean space R^n called an Interval Valued Function (IVF) i.e., $F(x) = F(x_1, x_2, ..., x_n)$ is a closed interval in R. The IVF F can also be written as $F(x) = [\underline{F}(x), \overline{F}(x)]$, where $\underline{F}(x)$ and $\overline{F}(x)$ are real-valued functions defined on R^n and satisfy $\underline{F}(x) \leq \overline{F}(x)$ for every $x \in R^n$. Let us consider the IVF F is differentiable at $x_0 \in R^n$ if and only if the real–valued functions $\underline{F}(x)$ and $\overline{F}(x)$ are differentiable at x_0.

Remark 2.2: Suppose A = $[\underline{a}, \overline{a}]$, B = $[\underline{b}, \overline{b}]$, then

1) F (A ≥ B) > 0 $\Leftrightarrow \overline{a} > \underline{b}$,

2) F (A > B) >0 $\Leftrightarrow \underline{a} > \underline{b}$ or $\overline{a} > \overline{b}$,

3) F (A ≤ B) >0 $\Leftrightarrow \underline{a} < \overline{b}$,

4) F (A < B) >0 $\Leftrightarrow \underline{a} < \underline{b}$ or $\overline{a} < \overline{b}$.

Definition 2.3: Let D denote the set of all closed bounded intervals on the real line R.
That is, D={[a,b]:a ≤ b, a and b are in R}.
Let A=[a,b] and B=[c,d] be in D. Then

(i) $A \oplus B$=[a+c,b+d] and

(ii) $A \otimes B$=[p,q] where p=minimum{ac,ad,bc,bd} and q=maximum{ac,ad,bc,bd}

3. Standard representations of FCTP and ITP

The FCTP can be described as a distribution problem in which m sources and n destination are involved. The product can be transported from each m sources to any of n destinations with associated cost of C_{ij} per unit. In addition a fixed charge of f_{ij} appears in the objective function if the associated variable means X_{ij} is positive. In a balanced FCTP it is assumed that the total amount of supplies in sources is equal to the sum of demand parameters in different destinations, but in real system problems this condition may not always hold;

$$\min \sum_{i=1}^{m}\sum_{j=1}^{n}(C_{ij}x_{ij} + f_{ij}y_{ij})$$

Subject to

$$\sum_{j=1}^{n} x_{ij} \leq a_i \quad i=1,\ldots,m$$

$$\sum_{i=1}^{m} x_{ij} \geq b_j \quad j=1,\ldots,n$$

$$x_{ij} \geq 0 \text{ for all } i=1,\ldots,m, j=1,\ldots,n$$

$y_{ij}=0$ if $x_{ij}=0$
$y_{ij}=1$ if $x_{ij}>0$

Where

$$\sum_{i=1}^{m} a_i \geq \sum_{j=1}^{n} b_j$$

Consider with all of parameters given in interval forms, and then the IP is formulated as follows:

$$\min \sum_{i=1}^{m}\sum_{j=1}^{n}([C_{L_{ij}}, C_{R_{ij}}]x_{ij} + [f_{L_{ij}}, f_{R_{ij}}]y_{ij})$$

Subject to

$$\sum_{j=1}^{n} x_{ij} \leq_1 [a_{L_i}, a_{R_i}] \text{ i=1,........,m}$$

$$\sum_{i=1}^{m} x_{ij} \geq_1 [b_{L_i}, b_{R_i}] \qquad \text{j=1,.......,n}$$

$x_{ij} \geq 0$ for all i=1,….m,j=1,….n

$y_{ij}=0$ if $x_{ij}=0$
$y_{ij}=1$ if $x_{ij}>0$

Where the inequality relations denoted by are defined as follows

$W \leq_1 [a,b] \equiv \exists z \in [a,b]; W \leq z,$

$W \geq_1 [a,b] \equiv \exists z \in [a,b]; W \geq z.$

3.1.Fully interval integer transportation problems

Consider the following fully integer transportation problem(P)

(P) Minimize $z=[z_1,z_2]=\sum_{i=1}^{m}\sum_{j=1}^{n}[c_{ij}, d_{ij}] \otimes [x_{ij}, y_{ij}]$

Subject to

$$\sum_{j=1}^{n}[x_{ij}, y_{ij}] = [a_i, p_i] \text{ i=1,2,.....m} \quad \text{------------(1)}$$

$$\sum_{i=1}^{m}[x_{ij}, y_{ij}] = [b_j, q_j] \qquad \text{j=1,2,.....n} \quad \text{-------------(2)}$$

$x_{ij} \geq 0, y_{ij} \geq 0$ i=1,2,……m and j=1,2,…….n and are integers--------(3)

Where c_{ij} and $d_{ij}$for positive real numbers for all i and j, a_i and p_i are positive real numbers for all i and b_j and q_j are positive real numbers for all j.

Definition 3.1: The set {[x_{ij},y_{ij}] for all i=1,2,…..m and j=1,2,….n }is said to be a feasible solution of (P) if they satisfy equation (1)-(3).

Definition 3.2: A feasible solution {[x_{ij},y_{ij}] for all i=1,2,…..m and j=1,2,….n} of the problem(P) is said to an optimal solution of (P) if $\sum_{i=1}^{m}\sum_{j=1}^{n}[c_{ij}, d_{ij}] \otimes [x_{ij}, y_{ij}] \leq \sum_{i=1}^{m}\sum_{j=1}^{n}[c_{ij}, d_{ij}] \otimes [u_{ij}, v_{ij}]$,

For all feasible {[u_{ij},v_{ij}] for i=1,2,….m and j=1,2,….n

Let$m(Z)=m([z_1,z_2]);w(Z)=w([z_1,z_2]);m([c_{ij},d_{ij}])=t_{ij};w([c_{ij},d_{ij}])=s_{ij};m([x_{ij},y_{ij}])=m_{ij};w([x_{ij},y_{ij}])=w_{ij};m([a_i,p_i])=u_i;w([a_i,p_i])=v_i;m([b_j,q_j])=k_j$and $w([b_j,q_j])=h_j$

Theorem 3.1:

If the set {U_{ij}^*, for all I and j} is an optimal solution of the upper value transportation problem(U) of (P)

Where (U) Minimize u(z)= $\sum_{i-1}^{m}\sum_{j=1}^{n} t_{ij}u_{ij}$

Subject to

$$\sum_{j=1}^{n} u_{ij} = p_i \quad i=1,2,\ldots,m$$

$$\sum_{i=1}^{m} u_{ij} = g_j \qquad j=1,2,\ldots,n,$$

$u_{ij} \geq 0$, i=1,2,….m and j=1,2,…..n and the set {W_{ij}^*, for all I and j} is an optimal solution of the half – width transportation problem(W) of (P) where

(W) Minimize w(z)= $\sum_{i=1}^{m}\sum_{j=1}^{n} s_{ij}w_{ij}$

Subject to

$$\sum_{j=1}^{n} w_{ij} = q_i \quad i=1,2\ldots.m$$

$$\sum_{i=1}^{m} w_{ij} = h_j \qquad j=1,2\ldots\ldots n$$

$w_{ij} \geq o$, i=1,2,….m and j=1,2,….n

then the set of intervals {[u_{ij}^*-w_{ij}^*,u_{ij}^*+w_{ij}^*], for all i and j} is an optimal solution to the problem (P) provided [u_{ij}^*-w_{ij}^*,u_{ij}^*+w_{ij}^*], for all i=1,2,….m and j=1,2,….n are integers.

Proof:

Given {u_{ij}^*, for all i and j} and {w_{ij}^*, for all i and j} are feasible solution to the problems (U) and (W) respectively and from the equality conditions of two intervals.

We have to prove that the set of intervals {[u_{ij}^*-w_{ij}^*,u_{ij}^*+w_{ij}^*], for all i and j} is a feasible solution to the problem (P)

Let the set {[x_{ij},y_{ij}], for all i=1,2,….m and j=1,2,….n} be a feasible solution to the problem (P)

Then, by the equality relation conditions, we can conclude that the set {u({[x_{ij},y_{ij}], for all i=1,2,….m and j=1,2,….n} is a feasible solution to the problem (U) and the set {w([x_{ij},y_{ij}], for all i=1,2,….m and j=1,2,….n} is a feasible solution to the problem(W)

Therefore, the set {u_{ij}^*, for all i and j} and the set {w_{ij}^*, for all i and j} are optimal solution to the problem (U) and the problem(W) respectively. It implies that

$$\sum_{i=1}^{m}\sum_{j=1}^{n} t_{ij} u([x_{ij}, y_{ij}]) \geq \sum_{i=1}^{m}\sum_{j=1}^{n} t_{ij} u_{ij}^{*} \quad \text{---------(1)}$$

And

$$\sum_{i=1}^{m}\sum_{j=1}^{n} s_{ij} w([x_{ij}, y_{ij}]) \geq \sum_{i=1}^{m}\sum_{j=1}^{n} s_{ij} w_{ij}^{*} \quad \text{------------(2)}$$

Using (1) we have,

$$U(z) = u\left(\sum_{i=1}^{m}\sum_{j=1}^{n} [c_{ij}, d_{ij}] \otimes [x_{ij}, y_{ij}]\right)$$

$$= \sum_{i=1}^{m}\sum_{j=1}^{n} u([c_{ij}, d_{ij}])([x_{ij}, y_{ij}])$$

$$\geq \sum_{i=1}^{m}\sum_{j=1}^{n} t_{ij} u_{ij}^{*} \quad \text{---------------(3)}$$

Using (2), we get

$$W(z) = w\left(\sum_{i=1}^{m}\sum_{j=1}^{n} [c_{ij}, d_{ij}] \otimes [x_{ij}, y_{ij}]\right)$$

$$= \sum_{i=1}^{m}\sum_{j=1}^{n} w([c_{ij}, d_{ij}]) w([x_{ij}, y_{ij}])$$

$$\geq \sum_{i=1}^{m}\sum_{j=1}^{n} s_{ij} w_{ij}^{*} \quad \text{---------(4)}$$

From (3) and (4), we get

$$\sum_{i=1}^{m}\sum_{j=1}^{n} [c_{ij}, d_{ij}] \otimes [x_{ij}, y_{ij}]) \geq \sum_{i=1}^{m}\sum_{j=1}^{n} [c_{ij}, d_{ij}] \otimes \{[u_{ij}^{*} - w_{ij}^{*}, u_{ij}^{*} + w_{ij}^{*}])$$

Thus the set of intervals $\{[u_{ij}^{*}-w_{ij}^{*}, u_{ij}^{*}+w_{ij}^{*}]$,for all i and j} is an optimal solution of the problem(P)

Hence the theorem is proved.

4. Upper –width method:

Let us introduce, a new algorithm namely, upper –width method for finding an optimal solution to a fully interval integer transportation problem(P)

The Upper-width method proceeds as follows

Step-1:

Construct two independent transportation problems called upper value transportation problem(U) and half-width transportation problem(W) from the given problem(P)

Step-2:

Solve the problem(U) using a transportation algorithm. Let $\{u_{ij}^*$, for all i and j$\}$ be an optimal solution of the problem(U)

Step-3:

Solve the problem(W) using any transportation algorithm. Let $\{w_{ij}^*$, for all I and j$\}$ be an optimal solution of the problem(W)

Step-4:

The optimal solution of the given problem(P) is $\{[u_{ij}^*-w_{ij}^*, u_{ij}^*+w_{ij}^*]$, for all i and j$\}$ if

$[u_{ij}^*-w_{ij}^*, u_{ij}^*+w_{ij}^*]$, for all i and j are integers.

The upper-width method for solving fully integer transportation problem is illustrated by the following examples.

Example:

An Apple company produces a product in its three factories F_1,F_2 and F_3. The product will be sent to four destinations D_1,D_2,D_3 and D_4 from the three factories. Determine a shipping plan for the company from three factories to four destinations such that the total shipping cost should be minimum using the following numerical data obtained from the company.

The minimum supply from F_1,F_2 and F_3 are 7,17 and 16 respectively and the maximum supply from F_1,F_2 and F_3 are 9,21, and 18 respectively. The minimum demand for D_1,D_2,D_3 and D_4 are 10,2,13 and 15 respectively and the maximum demand for D_1,D_2,D_3 and D_4 are 12,4,15 and 17 respectively.

The unit shipping cost range from each supply point to each demand point is given below.

Table:

Unit shipping cost range from supply points to demand points

	D_1	D_2	D_3	D_4
F_1	[1,2]	[1,3]	[5,9]	[4,8]
F_2	[1,2]	[7,10]	[2,6]	[3,5]
F_3	[7,9]	[7,11]	[3,5]	[5,7]

Solution:

Table-1:

	D_1	D_2	D_3	D_4	supply
F_1	[1,2]	[1,3]	[5,9]	[4,8]	[7,9]
F_2	[1,2]	[7,10]	[2,6]	[3,5]	[17,21]
F_3	[7,9]	[7,11]	[3,5]	[5,7]	[16,18]
Demand	[10,12]	[2,4]	[13,15]	[15,17]	[40,48]

Table-2:

Upper-value TP (U) of the problem(P)

	D_1	D_2	D_3	D_4	supply
F_1	2	3	9	8	9
F_2	2	10	6	5	21

F_3	9	11	5	7	18
Demand	12	4	15	17	48

Table-3:

Half –width TP(W) of the problem(P)

	D_1	**D_2**	**D_3**	**D_4**	**Supply**
F_1	0.5	1	2	3.5	1
F_2	0.5	1.5	2	1	2
F_3	1	1	1	1	1
Demand	1	1	1	1	4

The given problem modified as interval ITP as follows

An optimal solution to the problem(U) is $u_{11}^{*}=9$, $u_{21}^{*}=3$,$u_{24}^{*}=14$,$u_{33}^{*}=15$,$u_{34}^{*}=3$ and

An optimal solution to the problem(W) is $w_{11}^{*}=1$,$w_{12}^{*}=1$,$w_{21}^{*}=1$,$w_{23}^{*}=1$,$w_{24}^{*}=1$,$w_{33}^{*}=1$

Therefore, an optimal solution to the given transportation problem (P) is

$[x_{11},y_{11}]=[8,10]$,$[x_{21},y_{21}]=[3,3]$,$[x_{22},y_{22}]=[3,5]$,$[x_{24},y_{24}]=[13,15]$,$[x_{33},y_{33}]=[14,16]$,$[x_{34},y_{34}]=[3,3]$ with minimum interval transportation cost[128,252].

5. Conclusion

This paper focused on fixed charge TP when parameters are vague in the nature. In particular, all parameters are delivered in interval form. Different approaches which are considered while dealing with interval parameters have been investigated. A new method namely upper value method for computing an optimal solution to fully transportation problems has been proposed in this paper. A numerical example has been presented for demonstrating the solution procedure of the proposed method.

REFERENCES

[1] F.L.Hitchcock, Distribution of a product from several sources to numerous locations, J.Math. physics 20(1941).224-230

[2] G.Hadly, Linear Programming, Addison-wesley, publishing company, 1974.

[3] H.A.Taha, Operations Research and introduction, fifth edition, Macmillan Publishing Company, USA, 1992.

[4] S.Chanas, M.Delgado, J.L.Verdegay, M.A.Vila, interval and fuzzy extensions of classical transportation problems, Transportation planning and Technology 17(1993), 203-218.

[5] W.M.Hirsch, G.B Dantzing, The fixed charge problem, Naval Res.Log.Quart.15 (1968), 413-424.

[6] Sengupta.A, Pal.T.K, Interval valued transportation problem with multiple penalty factors, VU J. phys Sci. 2003; 9:71-81.

[7] A.Akilbasha, P.Pandian, G.Natarajan, "An innovative exact method for solving fully interval integer transportation problem", Informatics in Medicine Unlocked, 11(2018),95-99.

[8] M.R.Safi, A.Razmjoo, "Solving fixed charge transportation problem with interval parameters", Applied Mathematical Modelling, 37(2013), 8341-8347.

[9] S.M. AbulKalam Azad, Md. Bellel Hossain, Md. MizanurRahma,"An algorithmic approach to Solve transportation problems with the average total opportunity cost method", International Journal of Scientific and Research Publications, Volume 7(2017) 266 -270.